A Joint Space Formulation for Compliant Motion Control of Robot Manipulators

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Abstract—This paper presents a joint space formulation for robot manipulator’s hybrid motion/force control. The motivations come from 1) extending the previous work to general (either constrained or redundant) robots; and 2) improving the robustness against disturbances originated at the joint level. Contact geometry and closed-loop dynamics will be derived in this paper, also a joint space hybrid control scheme will be proposed. At the end, we show some simulation results to verify the applicability of our theory on a constrained (4-degree-of-freedom) robot WAM.

Index Terms—Compliant motion control, hybrid motion/force control, robot manipulator, joint space, projector.

I. INTRODUCTION

Many applications of robot manipulators require the end-effector to keep contact with some external environment, e.g., polishing, grasping etc. The general compliant motion control refers to those control schemes that actively maintain the contact through an extra force feedback loop. Among them, the hybrid motion/force control approach, which is firstly proposed by Raibert and Craig [1], aims to simultaneously control not only the position in the unconstrained degrees of freedom (DOFs) but the contact force in the constrained degrees of freedom as well. In a conventional hybrid control system, the workspace is decomposed into purely motion controlled directions and purely force controlled directions, and two parallel feedback loops are accordingly constructed for the separate control of motion and force.

The hybrid motion/force control has attracted roboticians a great amount of interests since early 1980s and there is already a rich body of literature on this topic referring to various issues including control algorithm [1]-[8], contact modelling [9][10], kinematic consistency [11]-[15], kinematic stability [16]-[18], dynamical decoupling [19][20] etc. Fig.1 illustrates the generic structure for most of the existing hybrid motion/force control schemes, which we think can further be roughly divided into the following four categories.

1) Joint space servoing without inverse dynamics

This stereotype appears in the pioneer work of hybrid control [1][2] (Block B/C and E/F are swapped here). The desired motion/force trajectories are compared with the actual measurements, then further projected onto their respective controlled subspaces through the so-called “selection matrices” (Block A,D). The filtered signals in the operational space will be transformed into the joint space (Block C,F), where some PID-like controllers are implemented (Block B,E), by using \( J^{-1} \) or \( J^\top \) (J is the manipulator Jacobian [21]).

2) Operational space servoing without inverse dynamics

2) is similar to 1) except that the servoing units operate before the transformations, thus are formulated in the operational space, i.e., the outputs of the PID-like controllers are 6-dim wrenches, which will later be converted into the joint drive torques [6].

3) Operational space servoing with inverse dynamics

3) is similar to 2) apart from Block B, in which the PID-like controller produces a 6-dim acceleration instead, in other words, its parameters become time constants rather than stiffness coefficients in 2). The calculated acceleration will be multiplied by the robot’s operational space inertia matrix to ultimately get the 6-dim drive wrench [3].

4) Constraint space servoing with inverse dynamics

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4) deals with the filtering and servoing in a slightly different way as 1), 2) and 3). The motion/force signals are mapped into their corresponding lower-dimensional constraint spaces (also called the local spaces), and this process in fact has the same effect as the projection step of other approaches. Accordingly, the servoing is performed at the downgraded level, and the results will later be transformed up to the operational space and eventually to the joint space [4][5][7].

We find most of the hybrid motion/force control algorithms in the literature have implemented both the path planning and the signal filtering in the operational space, which is clearly a reasonable choice because the contact is naturally easier described there. However, if the robot does not have exactly 6 DOFs, the implementation of the operational space oriented analysis will become a bit difficult or awkward, e.g., the operational space oriented decoupling, asymptotic stability etc; Section IV shows the control algorithm and discusses a few issues such as dynamical loop dynamics. Next we introduce the contact model, first formulated to model the contact task and the robot kinematics. A numerical optimal path planning method for robots subject to motion constraints has been developed recently [23], which can be used to calculate the desired joint trajectories in this scenario. The rest of this paper is organized as follows: Section II formulates the contact model and derives the robot closed-loop dynamics; Section III presents our joint space hybrid control algorithm and discusses a few issues such as dynamical decoupling, asymptotic stability etc; Section IV shows the simulation results of applying our control law on a 4-DOF experimental robot manipulator WAM.

II. CONTACT MODEL AND ROBOT CLOSED-LOOP DYNAMICS

This section first reviews the concept of the dual vector spaces $\mathcal{M}^6$ ($\mathcal{M}^6$) and $\mathcal{F}^6$ ($\mathcal{F}^6$), which respectively represent for velocities/accelerations and forces in the rigid body (robot) dynamics. Next we introduce the contact model, first formulated in the operational space, then migrated to the joint space, and derive the robot manipulator’s closed-loop dynamics. Our discoveries and the known results will be compared at the end of this section.

A. Geometry of Constrained Rigid Body (Robot) Systems

Consider a single rigid body which can translate and rotate freely in a 3-dim workspace. The configuration space $\mathcal{Q}$ for such an object is then the Euclidian group $SE_3$. By right translation, the tangent space to $\mathcal{Q}$ at any $p \in \mathcal{Q}$ can be identified with the Lie algebra $se_3$, which is a 6-dim real vector space, sometimes denoted here by $\mathcal{M}^6$. The space $\mathcal{M}^6$ consists of all possible rigid body velocity vectors $\dot{v}$ of the free-flying object [21].

Meanwhile, we can introduce the dual space of $\mathcal{M}^6$, denoted by $\mathcal{F}^6$, which can be identified with the set of linear functionals $F: \mathcal{M}^6 \rightarrow \mathbb{R}$. If we represent $F$ as a 6-dim column vector $\hat{f}$, then the canonical evaluation map $F(\dot{v})$ is given as

$$F: \mathcal{M}^6 \rightarrow \mathbb{R}, \dot{v} \mapsto \hat{f}^T \dot{v},$$

where $\hat{f}$ can be regarded as any 6-dim wrench (generalized force) [21] applied on or by the rigid body object [7][15].

Now suppose we have a $q$-joint robot manipulator constrained by mechanical stops, i.e., the joint position $q$ is an element of the direct product of $q$ open intervals. Ideally, the set of joint velocities $\dot{q}$ can be identified with the $q$-dim real vector space $\mathcal{M}^q$, and any joint acceleration $\ddot{q} \in \mathcal{M}^q$ as well. Similar to the analysis above, $\mathcal{M}^q$’s dual space $\mathcal{F}^q$ can be considered as the set of all joint torques $\tau$.

Between the corresponding dual spaces, we can study a linear map, denoted by $H^q_j: \mathcal{F}^q \rightarrow \mathcal{M}^q$ (the subscript $j$ or $o$ in this paper stands for the joint or operational space); and if it is invertible, we call the inverse $H^o_j$. For example, we can choose the robot inertia $H$ as $H^o_j$

$$H: \mathcal{M}^q \rightarrow \mathcal{F}^q, \dot{q} \mapsto \tau.$$  

The linear map $H$ is well known to be symmetric and positive definite, in particular, it is invertible. Likewise, we can define maps in the operational space, $H^o_o: \mathcal{F}^o \rightarrow \mathcal{M}^o$ or $H^o_o: \mathcal{M}^o \rightarrow \mathcal{F}^o$.

To relate these properties in the joint and operational space, we need to employ the manipulator Jacobian, which is usually defined as the map from the joint to the end-effector velocities. More specifically, if the above mentioned rigid body is the end-effector of the $q$-joint robot manipulator, we have the following relationship

$$\dot{v} = J(q) \dot{q},$$
where $J(q) \in \mathbb{R}^{6 \times q} : M^q \to M^6$, $\dot{q} \to \dot{\mathbf{v}}$. By duality, the transpose of $J$ can serve as the map between the corresponding dual spaces, i.e.,

$$\tau = J(q)^T \mathbf{f}.$$  \hspace{1cm} (4)

Combining Eqs.3 and 4 with 2, we get $H^T_{\alpha} = JH^{-1}J^T$; and if $J$ is invertible, $H^T_{\alpha} = J^{-1}H J^{-1}$ holds as well. However, we need to be aware that $H^T_{\alpha}$ does not always exist, e.g., if the robot has less than 6 DOFs or is at a singularity.

Next we model the force or motion constraints of the robot’s end-effector due to the contact as

$$\mathbf{f} = N\alpha, \ N^T \dot{\mathbf{v}} = 0,$$  \hspace{1cm} (5)

where $\alpha \in F^m$, and $F^m$ is the so-called constraint space (or local space). The matrix $N \in \mathbb{R}^{6 \times m}$ has full column rank, representing the linear map from $F^m$ to $F^6$. Moreover, it can be considered as a basis matrix for the $m$-dim subspace for all of the achievable contact forces, denoted by $N'_j$, in $F^6$. Note that the constraint $N^T \dot{\mathbf{v}} = 0$ defines the $(6 - m)$-dim subspace $T$ for all of the allowable velocities in $M^6$. If $T$ is the matrix representation of $T$, then $N^T T = 0$ (called reciprocity).

For example, a compliant motion task requires the manipulator’s end-effector to move on and maintain a single-point contact with a 2-dim plane (Fig.3). Here $n \in \mathbb{R}^3$ is the plane normal, $p \in \mathbb{R}^3$ is the linear position of the end-effector, both expressed in the inertial frame. Using the cross product in $\mathbb{R}^3$, in this situation,

$$\begin{bmatrix} n \\ p \times n \end{bmatrix}, \alpha \in \mathbb{R}.$$

Combining Eq.5 with Eqs.3 and 4, we get

$$\tau = J^T N \alpha = N_j \alpha', N^T J \dot{q} = N_j^T \dot{q} = 0,$$  \hspace{1cm} (7)

where $\alpha' \in F^{m'}$, $m' \leq m$, $N_j \in \mathbb{R}^{6 \times m'}$ also has full column rank. It happens that $m' < m$ if $J^T N$ is singular, i.e., $\text{Ker}(J^T) \cap N' \neq \{0\}$. In this case, $N_j$ can be obtained by performing a singular value decomposition on $J^T N$

$$J^T N = U \Sigma V^T,$$  \hspace{1cm} (8)

and extracting those column vectors, $m'$ in number, in $U$ corresponding to the $m'$ non-zero singular values. In the same way, $N_j$ defines the subspaces $N'_j$ and $T_j$ in $F^q$ and $M^q$ respectively.

Remember $H$ being symmetric positive definite. The $H^{-1}$-orthogonal subspace of $N'_j$, denoted by $N''_j$, or the $H$-orthogonal subspace of $T_j$, denoted by $T''_j$, can be given as

$$N''_j = \{ \tau_2 | r_1 H^{-1} \tau_2 = 0, \tau_1 \in N'_j \},$$

$$T''_j = \{ q_2 | q_1 H q_2 = 0, q_1 \in T_j \}.$$  \hspace{1cm} (9)

In Fig.2, the connections among those spaces and constraint spaces are illustrated.

According to Fig.2, we can build up the “dynamical projectors” $\Phi_{fj}, \Phi_{fj}'$, $\Phi_{mj}$ and $\Phi_{mj}'$ as follows (apart from $j$ and $\alpha$, the subscript $m$ or $f$ stands for motion or force; and $I_n$ in this paper denotes the $n \times n$ identity matrix). The meaning of this name will become clearer in the next subsection.

$$\Phi_{fj} : F^q \to N'_j, \ \Phi_{fj}' : T'_j \to \mathbb{R},$$

$$\Phi_{mj} : M^q \to T'_j, \ \Phi_{mj}' = H^{-1} N_j (N_j^T H^{-1} N_j)^{-1} N_j^T H^{-1},$$

$$\Phi_{fj} = H \Phi_{mj}' H^{-1}, \ \Phi_{fj}' = \Phi_{mj}.$$  \hspace{1cm} (10)

Because $H$ is symmetric positive definite, it is easily seen that

$$\Phi_{fj}^2 = \Phi_{fj},$$

$$\langle \tau_1, \Phi_{fj} \tau_2 \rangle_{H^{-1}} = \langle \Phi_{fj} \tau_1, \tau_2 \rangle_{H^{-1}},$$  \hspace{1cm} (15)

where

$$\langle \cdot, \cdot \rangle_{H^{-1}} : F^q \times F^q \to \mathbb{R},$$

$$\langle \tau_1, \tau_2 \rangle_{H^{-1}} \mapsto \tau_1 H^{-1} \tau_2.$$  \hspace{1cm} (16)

Similar properties as Eqs.15,16 hold for three other dynamical projectors as well, e.g., $\Phi_{mj}$ is an orthogonal projector being self-adjoint but with respect to $\langle \cdot, \cdot \rangle_H$.

B. Equations of Motion

Recall the conventional robot dynamics equation together with the motion constraints Eq.7, the robot closed-loop dynamics can be expressed as the system

$$H(q) \ddot{q} + C(q, \dot{q}) + \tau_c = \tau,$$

$$\dot{N}_j^T \ddot{q} + \ddot{N}_j^T \dot{q} = 0,$$  \hspace{1cm} (18)

where $C(q, \dot{q})$ is the gravity combined with the Coriolis term. Moreover, $\tau_c = N_j \alpha'$ is the corresponding joint torque to the contact force applied from the robot to the environment, and $\tau$ is the entire joint drive torque.

From Eq.18, we can solve for $\alpha'$ and $\ddot{q}$ as

$$\alpha' = (N_j^T H^{-1} N_j)^{-1} \left( N_j^T \ddot{q} + N_j^T H^{-1} (\tau - C) \right),$$

$$\ddot{q} = H^{-1} \left( (I_q - N_j (N_j^T H^{-1} N_j)^{-1} N_j^T H^{-1}) (\tau - C) \right.$$

$$\left. - N_j (N_j^T H^{-1} N_j)^{-1} N_j^T \dot{q} \right).$$  \hspace{1cm} (19)
After some algebraic manipulation, Eq.19 can be rewritten as
\[ H\Phi_{mj}\dot{q} = \Phi'_{fj}(\tau - C), \]
\[ H\Phi'_{mj}\dot{q} = \Phi'_{fj}(\tau - C) - \tau_c. \]  (20)

From Eq.20, we can see that by using the dynamical projectors \( \Phi_{fj}, \Phi'_{fj}, \Phi'_{mj} \) and \( \Phi_{mj} \), the closed-loop dynamics can be analyzed as two decoupled subsystems.

When \( J^TN \) has full rank, \( N_c \) can be substituted by \( J^TN \), as a result, Eq.20 will transform into Yoshikawa's formulation [4].

Furthermore, as we already mentioned before, if the robot manipulator has 6 DOFs and is free from singularities, i.e., \( J \) is invertible, there exist the following transformations
\[ H_o^\prime = J^{-1}HJ^{-1} = H_o, \]
\[ H_o^{\prime\prime} = JH^{-1}J^T = H_o^{-1}, \]  (21)
where \( H_o \) is called the operational space inertia matrix, and
\[ C(q, \dot{q}) = J^TC_o(p, \dot{v}) + H^{-1}J\dot{q}, \]  (22)
where \( C_o \) combines the operational space gravity and Coriolis term [3].

Plug Eqs.21, 22, \( N^\prime_j = N^TJ + N\dot{J} \) and \( \dot{a} = J\dot{q} + \dot{J}q \) (\( a \) being the derivative of \( \dot{v} \) with respect to time) into Eq.18, we then get a similar formulation of the robot closed-loop dynamics but in the operational space:
\[ H_o(p)\dot{a} + C_o(p, \dot{v}) + \dot{f}_c = \dot{f}, \]
\[ N^\prime\dot{a} + N\dot{v} = 0, \]  (23)
where \( \dot{f}_c \) is the contact force and \( \dot{f} \) is the 6-dim drive wrench.

The system response of Eq.23 can be brought to a similar form as Eq.20, which is consistent with the Lagrangian formulation in [7]:
\[ H_o\Phi_{mo}\dot{a} = \Phi'_{fo}(\dot{f} - C_o), \]
\[ H_o\Phi'_{mo}\dot{a} = \Phi'_{fo}(\dot{f} - C_o) - \dot{f}_c, \]  (24)
where
\[ \Phi'_{fo} : \mathbb{R}^6 \rightarrow N', \Phi_{fo} = N(\mathbb{N}^T H_o^{-1}N)^{-1}\mathbb{N}^T H_o^{-1}, \]  (25)
\[ \Phi'_{fj} : \mathbb{R}^6 \rightarrow N', \Phi'_{fo} = I_6 - \Phi_{fo}, \]  (26)
\[ \Phi'_{mj} : \mathbb{R}^6 \rightarrow T', \Phi'_{mo} = H_o^{-1}N(\mathbb{N}^T H_o^{-1}N)^{-1}\mathbb{N}^T, \]  (27)
\[ \Phi_{mo} : \mathbb{R}^6 \rightarrow T, \Phi_{mo} = I_6 - \Phi'_{mo}. \]  (28)
\[ \Phi_{fo} = H_o\Phi'_{mo}H_o^{-1}, \Phi_{fo} = H_o\Phi'_{mo}H_o^{-1}. \]  (29)
Again, \( N' \) and \( T' \) are defined by \( H_o \) and \( H_o^{-1} \) in the same manner as in Eq.9.

Finally, if comparing Eqs.25-28 with Eqs.10-13, we will find
\[ \Phi_{fj} = J^T\Phi_{fo}J^{-T}, \Phi'_{fj} = J^T\Phi'_{fo}J^{-T}, \]  (30)
\[ \Phi'_{mj} = J^{-1}\Phi'_{mo}J, \Phi_{mj} = J^{-1}\Phi_{mo}J. \]  (31)

### III. Hybrid Motion/Force Control Scheme Formulated in Joint Space

Based on the robot closed-loop dynamics (Eq.20), we move on to the joint space hybrid motion/force control scheme design in this section. The open-loop and the closed-loop control laws will be proposed one after another, and a key component called "kinematic projector" will be introduced and explained in detail. Also, during the development of our control algorithm, we will discuss two important issues of the system response: dynamical decoupling and asymptotic stability.

#### A. Open-loop Control Law Design and Dynamical Decoupling

Assuming the robot dynamics model is available, now consider the following open-loop control law
\[ \tau = H\dot{q}_d + C + J^T\dot{f}_{cd}, \]  (32)
where \( \dot{q}_d \) and \( \dot{f}_{cd} \) are the desired joint acceleration and contact force, respectively.

If \( J^T\dot{f}_{cd} \in N'_j \), inserting Eq.32 into Eq.20, we get
\[ H\Phi_{mj}\dot{q} = H\Phi'_{mj}\dot{q}_d, \]
\[ J^T\dot{f} + H\Phi'_{mj}\dot{q} = H\Phi'_{mj}\dot{q}_d + J^T\dot{f}_{cd}. \]  (33)

Eq.33 can be regarded as the joint space version of the existing result of dynamical decoupling analysis [19][20], which shows the independence between the component of \( \dot{a} \) in \( T \) and \( \dot{f}_c \) in \( N \) attributed to an operational space hybrid control law equivalent to Eq.32.

Now if we make \( J^T\dot{f}_{cd} \) and \( \dot{q}_d \) both comply with the contact model, i.e., \( J^T\dot{f}_{cd} \in N'_j \) and \( N'_j \dot{q}_d - N'_j \dot{q}_d = 0 \) as well, then
\[ N'_j \dot{q}_d = N'_j \dot{q}_d = N'_j \dot{q}_d, \]  (34)
which leads to
\[ \Phi'_{mj}\dot{q}_d = \Phi'_{mj}\dot{q}_d. \]  (35)

Finally, from Eqs.33,35, a "genuine" dynamical decoupling form between motion and force can be shown as
\[ \dot{q} = \dot{q}_d, \]
\[ J^T\dot{f}_c = J^T\dot{f}_{cd}. \]  (36)

#### B. Closed-loop Control Law Design and Stability Analysis

As disturbances are usually inevitable in real life robotic systems, it is always essential to implement some servoing mechanisms to improve a control law’s robustness. Consider combining a motion PD-controller and a force I-controller with Eq.32, our closed-loop control algorithm is finally designed as
\[ \tau = H(\dot{q}_d + \Omega_{mj}K_v(\dot{q}_d - \dot{q}) + \Omega_{mj}K_p(q_d - q)) + (J^T\dot{f}_{cd} + \Omega_f J_k \int J^T(\dot{f}_{cd} - \dot{f}_c) dt) + C, \]  (37)
where \( K_p, K_v \) and \( K_i \) are the \( (q \times q) \)-dim coefficient matrices for the joint space PD/I-controllers. \( \Omega_f \) and \( \Omega_{mj} \) are named kinematic projectors (in contrast to dynamical projectors, Eqs.10-13) which are used to filter out the noise incompatible
Fig. 4. Block Diagram of Joint Space Hybrid Motion/Force Control Scheme

with the contact model from the original force or motion signal. Details will be presented next. Fig.4 illustrates the general data flow of Eq.37. Comparing with other hybrid motion/force control schemes, our design may be summarized as “joint space servoing, filtering with inverse dynamics”.

Recall how the dynamical projectors are derived in the last section, e.g., $\Phi_{mj} : M^q \rightarrow T_j$ (Eq.13), where $T_j$ directly comes from the contact and robot kinematic models (Eq.7), and its complement $T_j'$ is defined from $T_j$ together with the robot inertia $H$ (Eq.9). The joint space dynamical projector can be transformed into the operational space (Eq.30.31) when the Jacobian $J$ is invertible, in other words, $H_o$ exists.

The kinematic projectors on the contrary are sourced from kinematic models only. $\Omega_f$ and $\Omega_{mj}$ in Eq.37 are actually derived from the operational space projectors, e.g., ”selection matrices” [1] or ”kinestate filters” [11], and function in a similar way as them but in the joint space.

Let’s repeat the original analysis of the motion constraints (Eq.5): when the end-effector is in contact with the environment, $M^o$ or $F^o$ consequently splits into $T$ and $T'$ or $N$ and $N'$, whose matrix forms are $T$, $T'$, $N$ and $N'$ respectively. Different from Eq.9, $T'$ or $N'$ here are not the dual subspaces of $N'$ or $T$ in $M^o$ or $F^o$ respectively with regard to the linear map of $H^o_o$ or $H^o_o$, but are the constrained subspaces of $\nu$ or $\tilde{f}$ due to the contact. $T$, $T'$, $N$ and $N'$ will satisfy [19]

\begin{align}
N^T T &= 0, \quad N^T T' = 0, \quad (38) \\
N'^T T &= I_{6-m}, \quad N'^T T' = I_m. \quad (39)
\end{align}

Since $T$ and $T'$ are not necessarily $I_n$-orthogonal, neither are $N$ and $N'$ [12], the operational space kinematic projector $\Omega_{mo} : M^o \rightarrow T$ need to be given as [25]

\begin{align}
\Omega_{mo} = [T \mid T'] \begin{bmatrix} I_{6-m} & 0 \\ 0 & 0 \end{bmatrix} [T \mid T']^{-1}. \quad (40)
\end{align}

Now we will try to find $\Omega_{mo}$’s equivalent $\Omega_{mj}$ in the joint space. Firstly, we define $\tilde{J} : Ker(J) \rightarrow \text{Image}(J)$, such that

\begin{align}
\tilde{J}q = Jq, \quad \forall q \in Ker(J), \quad (41)
\end{align}

and $\tilde{J}$ is a bijection. Also, we name

\begin{align}
M_1 := T \cap \text{Image}(J), \quad M_2 := T' \cap \text{Image}(J), \quad (42)
\end{align}

$M_1$, $M_2$, $Q_1$ and $Q_2$ are vector spaces, and $\text{Image}(J) = M_1 \oplus M_2$, $\text{Ker}(J)^\perp = Q_1 \oplus Q_2$ (see Fig.5).

\begin{align}
Q_1 := \tilde{J}^{-1}(M_1), \quad Q_2 := \tilde{J}^{-1}(M_2). \quad (43)
\end{align}

$\tilde{J}$, $\tilde{J}^{-1}$, $\text{Ker}(J)^\perp$, $\text{Image}(J)$, $\text{Image}(J)^\perp$ and $\text{Image}(J)$ are defined in Fig.5. Division of $\tilde{M}^q$ and $\tilde{M}^o$ for Kinematic Projector Setup

**Theorem:** The projector $\Omega_{mj}$ mapping $\tilde{M}^q$ onto $Q_1 \oplus \text{Ker}(J)$ along $Q_2$ and its complementary projector $\Omega_{mj}'$, with the property that

1) $\Omega_{mj}(q_1 + q_0) = q_1 + q_0$, $\forall q_1 \in Q_1, \forall q_0 \in \text{Ker}(J)$;
2) $\Omega_{mj}q_2 = 0$, $\forall q_2 \in Q_2$;
3) $I_q - \Omega_{mj} = \Omega_{mj}'$.

The first step is because $q_0 \in \text{Ker}(J)$, i.e., $Jq_0 = 0$. The second step comes from $q_1 \in Q_1$, so that $Jq_1 = Jq_1 \in M_1 \in T$, and $\Omega_{mo}(Jq_1) = Jq_1$. The third step is the result of $q_2 \in Q_2$, then $\Omega_{mo}(Jq_2) = 0$. The second step is because $q_2 \in \text{Ker}(J)^\perp$ and $(I_q - J^\dagger)$ projects $M_q$ onto $\text{Ker}(J)$ along $\text{Ker}(J)^\dagger$.

\begin{align}
\Omega_{mj}q &= (I_q - J^\dagger)q_2 \\
&= 0.
\end{align}

The first step is the result of $q_2 \in Q_2$, then $\Omega_{mo}(Jq_2) = 0$. The second step is because $q_2 \in \text{Ker}(J)^\dagger$ and $(I_q - J^\dagger)$ projects $M_q$ onto $\text{Ker}(J)$ along $\text{Ker}(J)^\dagger$.

\begin{align}
\Omega_{mj}' &= J^\dagger(\Theta_\text{mo} - \Theta_\text{mj})J \\
&= J^\dagger(\Theta_\text{mo} - \Theta_\text{mj})J.
\end{align}

The force kinematic projectors can be constructed in the same way as $\Omega_{mj}$, $\Omega_{mj}'$. Here we present their expressions straightforward without proof.

\begin{align}
\Omega_{fj} &= J^T \Theta_{fo}(J^T)^\dagger + (I_q - J^\dagger(J^T)^\dagger), \quad (46) \\
\Omega_{fj}' &= J^T \Theta_{fo}'(J^T)^\dagger, \quad (47)
\end{align}
where \( \Omega_f = [\mathbf{N} \, | \, \mathbf{N}'] \begin{bmatrix} \mathbf{I}_m & 0 \\ 0 & 0 \end{bmatrix} [\mathbf{N} \, | \, \mathbf{N}']^{-1} \), and \( \Omega_f' = \mathbf{I}_6 - \Omega_f \).

An and Hollerbach [16] discovered that the traditional hybrid control scheme [1] without the robot dynamics compensation will become unstable in certain cases. In a later paper, Fisher and Mujtaba [17] imputed the instability to the joint space projectors, and proposed a substitutional formulation as
\[
\Omega_{mj} = (\mathbf{J} \Omega_{mo}) (\mathbf{J}^\top \Omega_{mo})^{-1}.
\]
This idea was followed by Doulgeri et al. [18].

Obviously, Eq.48 is not equivalent to Eq.44, and we suggest to employ the following simple example to make a comparison between those two expressions.

Suppose we have a Cartesian robot as in Fig.3 with a spherical wrist installed at its end-tip, then the Jacobian is simply a 6 \times 6 identity matrix. Obviously, in this situation, \( \Omega_{mo} \) and \( \Omega_{mj} \) should be identical. Plug \( \mathbf{J} = \mathbf{I}_6 \) into Eqs.44,48, we get
\[
\Omega_{mj} (\text{Ours}) = \mathbf{I}_6^\top \Omega_{mo} \mathbf{I}_6 + (\mathbf{I}_6 - \mathbf{I}_6^\top \mathbf{I}_6) = \Omega_{mo},
\]
\[
\Omega_{mj} (\text{Fisher’s}) = (\mathbf{I}_6 \Omega_{mo}) (\mathbf{I}_6^\top \Omega_{mo}) = \Omega_{mo} \Omega_{mo} = \mathbf{V} \Sigma^\top \mathbf{U}^\top \mathbf{U} \Sigma \mathbf{V}^\top.
\]

The second step in deriving Fisher’s projector utilizes a singular value decomposition where \( \mathbf{U}, \mathbf{V} \) are orthogonal matrices. As we can see, Fisher’s formulation drifts away from the original \( \Omega_{mo} \) (Eq.40) and ends up with an artificial orthogonal projector.

Now back to our hybrid control law (Eq.37), \( \Omega_{mj} \) or \( \Omega_{fj} \) is included so as to project the position displacement/velocity or force onto its own allowable degrees of freedom. Consequently, we will have
\[
\mathbf{N}_j^\top \left( \Omega_{mj} \mathbf{K}_p (\dot{\mathbf{q}}_d - \hat{\mathbf{q}}) + \Omega_{mj} \mathbf{K}_p (\mathbf{q}_d - \mathbf{q}) \right) = 0,
\]
\[
\left( \Omega_{fj} \mathbf{K}_i \int \mathbf{J}^\top (\hat{\mathbf{f}}_cd - \hat{\mathbf{f}}_c) \, dt \right) \in \mathcal{N}_j.
\]

Moreover, if the desired motion and force trajectories are planned wisely in a sense of fitting the contact model, then
\[
\mathbf{N}_j^\top \ddot{\mathbf{q}}_d = \mathbf{N}_j^\top \ddot{\mathbf{q}}_d \simeq \mathbf{N}_j^\top \hat{\mathbf{q}},
\]
\[
\mathbf{J}^\top \hat{\mathbf{f}}_cd \in \mathcal{N}_j.
\]

As a result, the overall feedback control input meets the requirement of realizing the dynamical decoupling (Eq.36), and the system response will ultimately become
\[
\ddot{\mathbf{q}} = \ddot{\mathbf{q}}_d + \Omega_{mj} \mathbf{K}_p (\dot{\mathbf{q}}_d - \hat{\mathbf{q}}) + \Omega_{mj} \mathbf{K}_p (\mathbf{q}_d - \mathbf{q}),
\]
\[
\mathbf{J}^\top \hat{\mathbf{f}}_c = \mathbf{J}^\top \hat{\mathbf{f}}_cd + \Omega_{fj} \mathbf{K}_i \int \mathbf{J}^\top (\hat{\mathbf{f}}_cd - \hat{\mathbf{f}}_c) \, dt.
\]

Define \( e_m := \mathbf{q}_d - \mathbf{q} \) and \( e_f := \mathbf{J}^\top \hat{\mathbf{f}}_cd - \mathbf{J}^\top \hat{\mathbf{f}}_c \), the above equation can further be rewritten as
\[
e_m + \Omega_{mj} \mathbf{K}_p e_m + \Omega_{mj} \mathbf{K}_p e_m = 0,
\]
\[
e_f + \Omega_{fj} \mathbf{K}_i \int e_f \, dt = 0. \tag{56}
\]
As long as \( \Omega_{mj} \mathbf{K}_p, \Omega_{mj} \mathbf{K}_p \) and \( \Omega_{fj} \mathbf{K}_i \) are chosen to be positive definite, both motion and force subsystems will enjoy asymptotic stability.

IV. SIMULATION AND FUTURE WORK

WAM is a 4-joint robot manipulator with human-like kinematics (see Fig.6). Viewed in the Cartesian space, WAM’s 4-DOFs correspond to its end-effector moving to an arbitrary 3D position within its reachable space and rotating about the axis from its base to end-tip (1-DOF in orientation).

![Fig. 6. 4-DOF Robot Manipulator WAM](image)

![Fig. 7. Proposed Compliant Motion Task for WAM](image)

Now suppose WAM is required to move its end-effector on a horizontal plane (see Fig.7), then such a single-point contact will divide WAM’s workspace into a combination of 3-DOF
motion and 1-DOF force, i.e., in the compliance (task) frame \([10]\) \(\{x, y, z\}\), 2-DOF translation in the \(x\)-\(y\) plane plus 1-DOF rotation; and a linear contact force along \(z\)-axis. For example, the compliant motion task illustrated in Fig.7 makes WAM’s end-tip to traverse an S-shape path with its body swinging back and forth; in the meantime, a fluctuant magnitude contact force can possibly be applied against the environmental surface as well.

An experiment on WAM of applying our joint space hybrid control algorithm for the proposed single-point contact task is under consideration. Figs.8-12 show the results of a preliminary simulation performed under Matlab 7.0 (The MathWorks, Inc. 2004). The better tracking performance of the motion controller as we see in Figs.8-11 and 12 is intrinsically due to the varied characteristics of the motion and force subsystems, i.e., disturbances of joint drive torques (simulated as white noises here) have more direct destructive effect on the eventual output of the contact force than that of the joint positions.

Furthermore, another reason accounting for the jagged force
response is the environment has been assumed as fixed (i.e., infinitely stiff) in the closed-loop dynamics simulation and hybrid control algorithm here. In the future, we wish to incorporate the contact dynamics as it is a critical factor for the controller’s capacity to reject the force disturbance [26]. Also, we will be interested in looking at the stability issue due to the transition between free and constrained motion, e.g., to reduce the influence of the impact force on the dynamical instability etc.

V. CONCLUSION

This paper presents a new scheme for the hybrid motion/force control of robot manipulators. Our contribution is to redo the formulation in the joint space so as to make the closed-loop dynamics analysis and the control algorithm design applicable to general robots. Also, the joint space control law allows us to implement different suitable control gains for each robot joint, which we think is a more practical and robust approach for real robotic systems whose disturbances are mainly originated at the joint level. An example of the compliant motion control for a 4-DOF robot WAM was presented and some simulation results have been shown at the end of the paper.

REFERENCES